جامعة بيرزيت

BIRZEIT UNIVERSITY

Mathematics Department

Math 330

Final Exam Summer Semester 07/08

Student name: .... ID no.: ...... sec........

QUESTION 1(45 Points): ***Solve in details then circle the correct answer***

(1) Use (compound)Simpson's rule with 4 subintervals to estimate π by approximating the integral $\int_0^1 \frac{4}{x^2+1}dx$

(a) $\frac{102573}{32650}$

(b) $\frac{5323}{1275}$

(c) $\frac{355}{113}$

(d) $\frac{8011}{2550}$

(e) None of the foregoing

$S_{1/3} = \frac{h}{3}[f_0 + 4f_1 + f_2]$ $h = \frac{1-0}{4} = \frac{1}{4}$

$S_{1/3} = \frac{1}{12}[4 + 4(3.765 + 3.2] = 3.14156866$

$\simeq$ (a)

(2) Let $S(x) = \begin{cases} ax^3 + \frac{15}{4}x + 2, & \text{for } 0 \le x \le 1 \\ \frac{3}{4}x^3 - bx^2 + \frac{33}{4}x + \frac{1}{2} & \text{for } 1 \le x \le 2 \end{cases}$ be cubic spline then

(a) $a = \frac{9}{2}$

(b) $a = \frac{3}{4}$

(c) $a = \frac{1}{2}$

(d) $a = \frac{7}{4}$

(e) None of the foregoing

$a = -\frac{3}{4}$

(3) Determine a, b and c such that the quadrature formula

$Q(f) = a\,f(0) + b\,f\left(\frac{5}{8}\right) + c\,f(1)$ Is exact for the integral $\int_0^1 f(x)x^2\,dx$. If $f(x)$ is a polynomial of degree ≤ 2, that is, is exact for $f(x) = 1, f(x) = x, f(x) = x^2$

Then a is

(a) $a = \frac{16}{76}$

(b) $a = \frac{7}{60}$

(c) $a = \frac{1}{300}$

(d) $a = \frac{2}{9}$

(e) None of the foregoing.

(4) For the nonlinear equation:

$$x - \tan(x) = 0,$$

Which fixed-point arrangement below will converge for all $x_0 \in [4,5]$?

(a) $x = \tan(x)$ discont. on [4, 5

(b) $x = \tan(x + \pi)$ dis

(c) $x = \pi + \tan^{-1}(x)$ $x \in [4,5] \Rightarrow g(x) = \tan^{-1} x \notin [4,5]$

(d) $x = \tan^{-1}(x)$

c $g(x) = \tan^{-1} x + \pi$ Cont on [4, 5

$4 \leq 4.467403 \leq g(x) \leq 4.514934 \leq 5$

$g'(x) = \frac{1}{x^2+1} < 1 \quad \forall x \in [4,5]$

7) use the 2- point Guass quadrature formula to approximate $\int_{-1}^{0}\int_{0}^{1}(x^2+y^4)dydx$

(a) 2/9
(b) 4/9
(c) 8/9
(d) 16/9
(e) 19/36

$\int_{-1}^{1} \frac{1}{x^2+4} dx$

$= f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{1}{\frac{1}{3}+4} + \frac{1}{\frac{1}{3}+4}$

$= \frac{6}{13}$

$= .462$

(8) Use the modified Euler's method:

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_n + h, y_n + hf(x_n, y_n))], y_0 = y(x_0)$$

With $h = 0.2$ to approximate $y(0.2)$ given that

$$y' = y - t^2 + 1; \quad y(0) = 0.5$$

(a) 1.168
(b) 0.834
(c) 1.152
(d) 0.826

(9) What is the Euler's method for the initial value problem?

$$y' = f(x,y), \quad a \le x \le b \quad ?$$
$$y(b) = y_0$$

(a) $x_n = a + nh,$
$y_{n+1} = y_n + hf(x_n, y_n);\ y_0 = y(b)$

$x_n = b - nh$

(b) $x_n = a + nh,$
$y_{n+1} = y_n - hf(x_n, y_n);\ y_0 = y(b)$

$y_{n+1} = y(x_n - h)$

$\simeq y_n - hf(x_n, y_n)$

(c) $x_n = b - nh,$
$y_{n+1} = y_n + hf(x_n, y_n);\ y_0 = y(b)$

(d) $x_n = b - nh,$
$y_{n+1} = y_n - hf(x_n, y_n);\ y_0 = y(b)$

QUESTION 2

(a) (10 Points) For $f(x) = (x-3)^4(x-2)$ what is rate of convergence of Newton method when its used to estimate the roots $x = 3$, $x = 2$ and the a asymptotic error constant for each.

root $= 3 \Rightarrow R = 1$, $A = \frac{4-1}{4} = \frac{3}{4}$

root $= 2 \Rightarrow R = 2$, $A = \left|\frac{f''(2)}{2f'(2)}\right| = \frac{8}{2} = 4$

(b) (10 Points) Show that Bisection method with an error$=\frac{b_0 - a_0}{2^{n+1}}$ converges liearly

$\lim \frac{e_{n+1}}{e_n} =$

$\lim\limits_{n \to \infty} \frac{b_0 - a_0}{2^{n+1}} \Big/ \frac{b_0 - a_0}{2^n}$

$= \lim \frac{2^n}{2^{n+1}} = \frac{1}{2} = A$

$\boxed{R = 1}$

QUESTION 3 (10 Points)

The Solid of revolution obtained by rotating the region under the curve $y = f(x) a \geq x b$

About the $x - ax$ has surface area given by

$$\text{Area} = 2\pi \int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx$$

Use the following to estimate the area y composite Trapizoidal rule?

x	1	2	3
$f(x)$	0.2	0.25	0.4
$f'(x)$	.05	.1	.15

$f'(x) = \frac{-3f_0 + 4f_{-1} - f_{-2}}{2h}$

$f'(x) = \frac{3f_0 - 4f_1 + f_2}{2h}$

$f'(1) = \frac{f(2) - f(1)}{1} = .25 - .2 = .05$

$f'(2) = \frac{f(3) - f(1)}{2(1)} = \frac{.4 - .2}{2} = .1$

$f'(3) = \frac{f(3) - f(2)}{1} = .4 - .25 = .15$

$f'(3) = \frac{3f(3) + f(1)}{2(1)}$

$\text{Area} = 2\pi \left[\int_1^2 g(x)\,dx + \int_2^3 g(x)\,dx \right]$

$= 2\pi \left[\frac{1}{2} \left[g(1) + 2g(2) + g(3) \right] \right]$

$= \pi \left[f(1)\sqrt{1+[f'(1)]^2} + 2f(2)\sqrt{1+[f'(2)]^2} + f(3)\sqrt{1+[f'(3)]^2} \right]$

$= \pi \left[.2\sqrt{1+(.05)^2} + 2(.25)\sqrt{1+(.1)^2} + .4\sqrt{1+(.15)^2} \right]$

$= \pi \left[.20025 + 2(.25124) + .4047 \right]$

$= 3.47909$

QUESTION 4 (10 Points)

Derive the two points forward to estimate $f'(x)$ and its error, and use it to esti

derivative of $f(x)=e^{-x}\cos x$ on [1,1.1]

$$f(x+h) = f(x) + h f'(x) + \frac{h^2 f''(c)}{2!}$$

$$\frac{f(x+h) - f(x)}{h} = f'(x) + h\frac{f''(c)}{2!}$$

$$f'(1) = \frac{f(1.1) - f(1)}{.1} = -0.477$$

$$f(x) \simeq P_1(x) = f_0 \cdot \frac{(x - x_1)}{x_0 - x_1} + f_1 \frac{(x - x_0)}{x_1 - x_0}$$

$$f'(x) = \frac{f_0}{x_0 - x_1} + \frac{f_1}{x_1 - x_0}$$

$$= \frac{f_0}{-h} + \frac{f_1}{h}$$

$$= \frac{f_1 - f_0}{h}$$

QUESTION 5

(1) (15 Points) Give the function $f(x) = \frac{1}{2-5x}$ and its data

~~(-1,0.200),(0,0.500),(0.5,2)~~ . $(-1, .143), (.1, 1.5)$ $(.5, -2)$

Compare the cost of estimating $P_2(2)$ by

(a) Solving system of 3×3 using Gaussian elimination. $P_2(x) = ax^2 + bx + c$

(b) Using Lagrange interpolation

(c) Using Newton interpolation

(a)

$a - b + c = .143$

$.01a + .1b + c = 1.5$

$.5a + .25b + c = -2$

$\boxed{G.E.}$ $c = 2.6422$

$b = -4.7565$

$a = -6.6557$

$\boxed{\text{cost} = 28}$

(b)

$$P_2(x) = \frac{.143(x-.1)(x-.5)}{(-1-.1)(-1-.5)} + \frac{1.5(x-(-1))(x-.5)}{(.1-(-1))(.1-(.5))}$$

$$+ \frac{-2(x-(-1))(x-.1)}{(.5-(-1))(.5-.1)}$$

$P_2(2) = -33.72$

$\boxed{\text{cost} = 26}$

$P_2(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1)$

$a_0 = .143$

$a_1 = f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{1.5 - .143}{.1-(-1)} = 1.234$ 3

$a_2 = f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, 1]}{x_2 - x_0} = -6.6$ 3 + 3

$P_2(2) = .143 + 1.234(2-(-1)) + (-6.6)(2-(-1))(2-.1) = -34.$

(4)+(3)

cost = [illegible]

round - error has the bound $|e_k| \le 5 \times 10^{-6}$

x	$f(x) = \ln x$
2.8	1.02962
2.9	1.06471
2.998	1.09795
2.999	1.09828
3.0	1.09861
3.001	1.09895
3.002	1.09928
3.1	1.1314
3.2	1.16315

a) Approx. $f'(3)$ by using $\quad h = .1$

$$f'(x_0) \simeq \frac{-y_2 + 8y_1 - 8y_{-1} + y_{-2}}{12h} + E(f,h)$$

$$E_{total}(f,h) = E_{round}(f,h) + E_{trunc.}$$

$$= \frac{18\varepsilon}{12h} + \frac{h^4 f^{(5)}(c)}{30}$$

the best h $\Rightarrow h = \left(\frac{45}{4M}\right)^{\frac{1}{5}}$

$$\boxed{|E_t(f,h)| \le \left|\frac{18 \times 5 \times 10^{-6}}{12(.1)}\right| + \left|\frac{h^4 \max f^{(5)}(c)}{30}\right|}$$

$2.999 \le c \le 3.002$

$f'(x) = \frac{1}{x} = x^{-1}$

$f''(x) = -1\, x^{-2}$

$f'''(x) = 2\, x^{-3}$

$f^{(4)}(x) = -6\, x^{-4}$

$f^{(5)}(x) = 24\, x^{-5}$
$= \frac{24}{x}$

$M = f^{(5)}(c) = \frac{24}{2.999}$